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# proving things

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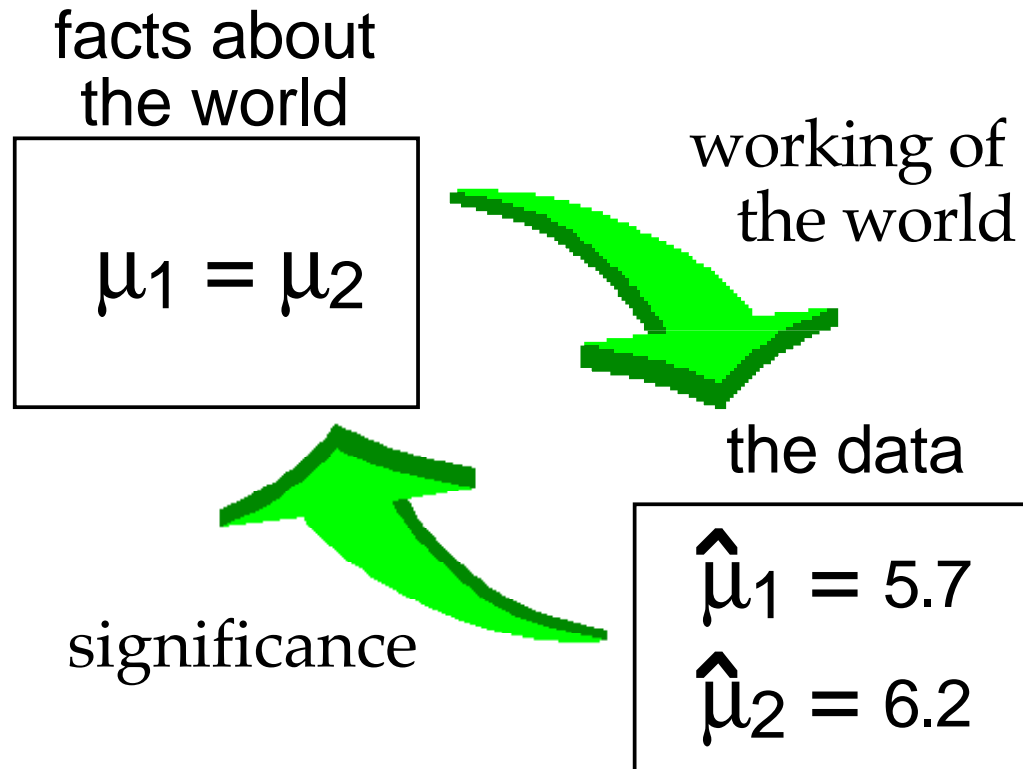
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- proving things are different  
importance & significance
- proving things are the same  
confidence intervals

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# working backwards again



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# deduction

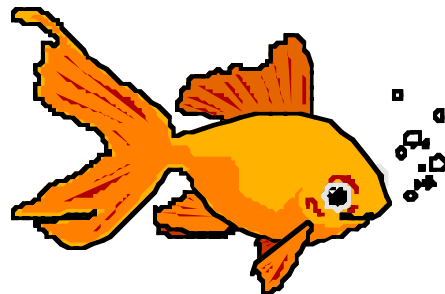
- all men are mortal
- Socrates is a man
- therefore Socrates is mortal

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# induction

- all men are mortal
- Socrates is mortal
- ? therefore Socrates is a man



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# scitsitats $\leftrightarrow$ statistics

we know:

if real means are different  
then sample means will be different

we measure:

sample means are different

? what to deduce

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# significance test

hypotheses:

$H_1$  – what want to show

$H_0$  – null hypothesis (to disprove)

- if  $H_0$  were true  
then observed effect is very unlikely  
therefore  $H_1$  is (likely to be) true

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# 5% significance level?

it says:

if  $H_0$  were true  
then probability observed effect  
happening by chance  
is less than 1 in 20 (5%)

so  $H_0$  is unlikely to be true  
and  $H_1$  is likely to be true

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# does not say

✘ probability of  $H_0$  is  $< 1$  in 20

✘ probability of  $H_1$  is  $> 0.95$

✘ effect is important  
i.e. significant in the real sense



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# all it says

- ✓ if  $H_0$  were true ...  
... effect is unlikely  
( prob. < 1 in 20)

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# proving differences

often:

$H_0$  – some things are equal

$H_1$  – they are different

statistically significance if

observed difference  $\gg$  random variation  
 $\hat{\mu} \gg \hat{\sigma} / \sqrt{n}$

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# proving differences

$$\frac{\hat{\mu}}{\hat{\sigma} / \sqrt{n}}$$

bigger ratio  $\Rightarrow$  smaller p  
(less likely by chance)

top and bottom both estimates  
(where 't' test comes in)

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# reasons for significance

$$\frac{\hat{\mu}}{\hat{\sigma} / \sqrt{n}}$$

$\hat{\mu}$  large – large difference (?important?)

$\hat{\sigma}$  small – small natural variation  
(e.g. natural science)

$n$  large – large sample

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# little things matter

$$\frac{\hat{\mu}}{\hat{\sigma}/\sqrt{n}}$$

what if small differences are important?  
(i.e.  $\hat{\mu}$  small)

$\hat{\sigma}$  fixed by the world

$\Rightarrow$  need large number of samples ( $n$ )

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# quick test

true or false?

- ① 5% significant  $\Rightarrow$   
     $H_0$  is false &  $H_1$  is true  
    i.e. things are different
  
- ② not significant  $\Rightarrow$   
     $H_0$  is true &  $H_1$  is false  
    i.e. things are the same

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# neither!!

- ① 5% significant  $\Rightarrow$   
things are different  
nearly true
- ② not significant  $\Rightarrow$   
things are the same  
NO! NO! NO!!!!

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# ① significant result

normally reason:

significant result  $\Rightarrow$   $H_0$  false  
things are different

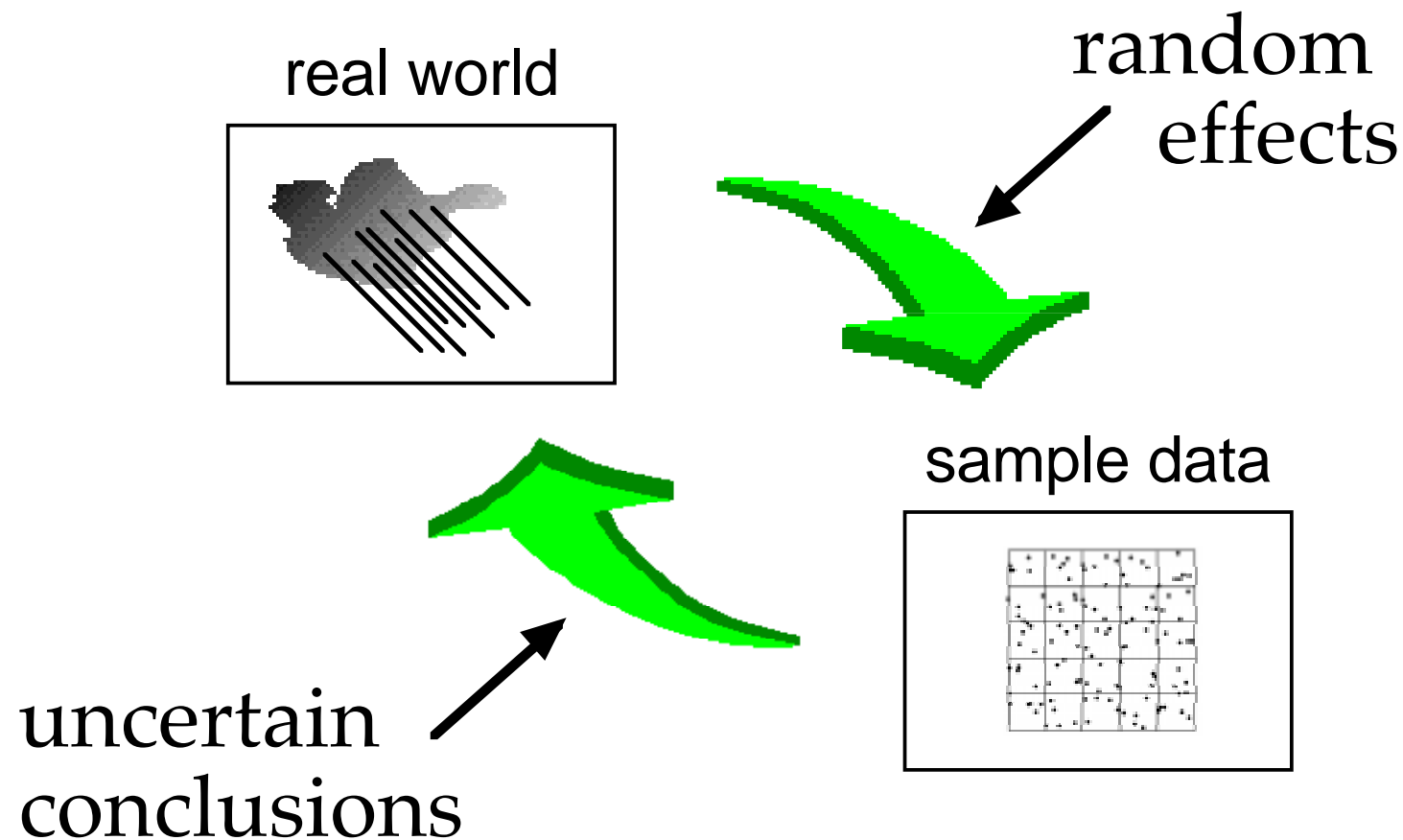
- 1 time in 20 you will be wrong  
e.g. conclude drug improves health  $p < 5\%$   
1 in 20 chance it kills everyone
- 1 in 20 significant results are false



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# statistical proof?



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# statistical proof

all you can do is:

- say something is true
- know how often you will be wrong  
(on average)
- choose how often you will be wrong!  
(i.e. choose the significance level)

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## ② non-significant result

can NEVER reason:

~~×~~ non-significant result  $\Rightarrow$   $H_1$  false ~~×~~  
things are the same

all you can say is:

$H_1$  is not statistically proven

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# reasons for non-significance

$$\frac{\hat{\mu}}{\hat{\sigma} / \sqrt{n}}$$

$\hat{\mu}$  too small – small effect  
(possibly important)

$\frac{\hat{\sigma}}{\sqrt{n}}$  too large – insensitive experiment  
not enough samples (n too small)  
too much variability ( $\hat{\sigma}$  too big)

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# lies ...

*prosecution:* and when did your dislike  
of the victim turn to hatred

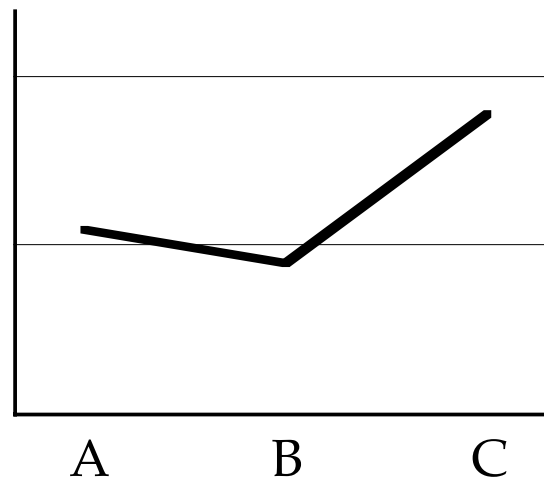
*defence:* objection

*prosecution:* withdrawn

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# damn lies ...



**Figure 5.3**

A	5.6
B	4.7
C	9.2

**Table 5.7**

The results of comparing the response times in the three conditions are shown in table 5.7 and figure 5.3. The response times in condition C were longer than those for A and B, but this difference was not significant ( $p=23\%$ ).

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# at InterCHI

*presenter:* (shows slide with graph)  
... not significant

*(at end of presentation)*

*questioner:* the results in the graph  
looked very interesting ...

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# proving equality

- non-significant – not proved different
- real difference may always be smaller than experimental error  
⇒ can never prove equality
- can put bounds on inequality



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# confidence interval

- bound on true value
- mean of data is 0.6  
95% confidence interval is  $[-0.7, 1.3]$
- says if you conclude:  
    real mean is in the range  $[-0.7, 1.3]$   
95% of the time you will be right!

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# counterfactuals

- 95% confidence interval is  $[-0.7, 1.3]$
- does not say:  
there is 95% probability that the  
real mean is in the range  $[-0.7, 1.3]$
- it either is or it isn't!
- all it says:  
95% probability that you are right

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# proven!

$H_0$ : no difference (real mean is zero)

experimental result: mean is 0.6

significance test: n.s. at 5%  
– so what!

95% confidence interval:  $[-0.7, 1.3]$

? is 1.3 is an important difference